

Methods III: Game Theory

Course Guide

2020–2021

A BEAUTIFUL EQUILIBRIUM

n players with strategies
 $s_1, s_2 \dots s_n$

pure
&
mixed!

$s_1^* \in s_1, s_2^* \in s_2 \dots s_n^* \in s_n$

are a NASH EQUILIBRIUM iff

$$\forall_i s_i^* = \operatorname{argmax}_{s_i} \operatorname{utility}_i(s_1^* \dots s_i \dots s_n^*)$$

no reason for any one to change



This document contains the guidelines and materials for the third year PPE course 'Methods of PPE III'. The document has three sections. The first provides key course details and contact information, along with a description of the course and its learning objectives. The second includes a schedule of teaching topics and readings as well as links to teaching materials. The third provides a list of assessment policies and marking guidelines as well as information on general John Stuart Mill college policies.

- Lectures:
 - Monday 15.30–17.15
 - Wednesday 09.00–10.45
- Seminars (math labs):
 - Tuesday 11.00–12.45
 - Thursday 11.00–12.45
- Lecturers:
 - Week 1–3: Marina Uzunova, MA (m.g.uzunova@vu.nl)
 - Week 4–6: Prof. Dr. Rene van den Brink (j.r.vanden.brink@vu.nl)
- Seminar instructor:
 - Marina Uzunova, MA (m.g.uzunova@vu.nl)
 - Weekly (Zoom) office hours: Friday at 10.00–11.00
 - Link for the office hours: <https://vu-live.zoom.us/j/95506005793?pwd=OUxHSnVpMVJUMTM2ZnVTSWZIOEJ1Zz09> (password: Methods3)

1. Course Description

Description

Conflict as well as cooperation are key features of our social world. Both follow from the interdependency of human actions. Conflicts arise when one person's interests clash with the interests of another person, whereas cooperation often emerges because individuals can jointly realize goods or benefits which they cannot realize on their own. Game theory is the systematic study of such interdependencies and it thus is crucial for our understanding of human interaction. The course provides a rigorous training in the tools of game theory and brings out the relevance of those tools for a wide variety of PPE-topics—ranging from the analysis of international relations to proposals for tax reform, and from negotiations between trade unions and employer organizations to the formation of governments. Game theoretic topics that will be covered include: extensive form and normal form games; incomplete information; equilibrium concepts;

evolutionary games; cooperative games (including applications such as voting and matching problems) and bargaining.

Aims

You acquire:

- Knowledge of main approaches in (cooperative and non-cooperative) game theory and their applications;
- The ability to describe and analyze a decision situation game-theoretically;
- The ability to assess the relevance of the use of game theory for the analysis of economic and political decision-making.

Form

Lectures and math labs.

Type of assessment

Weekly written tests (5 x 10%), final written exam (50%)

2. Course Schedule

Required readings are mandatory for each week's topic. The math labs will be based on the readings and on assignments. The numbers after 'Osborne' in the readings below refer to the respective chapter(s) (for example, 'Osborne 1–2' refers to chapters 1 and 2), or the respective section(s) (for example, 'Osborne 4.12' refers to section 4.12 in chapter 4) in the Osborne textbook.

Week 1: Game Theory: Introduction and Nash Equilibrium

The course starts with an overview of the field. The different areas of game theory are described and we explain how the resulting map of the territory relates to the course's set-up. The first four weeks cover non-cooperative theory. Lecture 1-1 recaps some of the material that was covered in the first year class on decision theory, including the analysis of some classic games in normal form and the Nash equilibrium. Lecture 1-2 discusses some applications of normal form games, such as the Cournot oligopoly, the Bertrand oligopoly, electoral competition and auctions.

Readings 1-1	Math Lab 1-1	Readings 1-2	Math Lab 1-2
Osborne 1–2	Exercises	Osborne 3.1–3.3, 3.5	Part 1: Test 1 (Material: 1-1) Part 2: Exercises

Week 2: Mixed Equilibria

Week 2 is devoted to the mixed Nash equilibrium in normal form games. Lecture 2-1 discusses randomization of strategies, the mixed Nash equilibrium concept, and its motivation. Also, the mixed Nash equilibrium will be put in the context of expected utility theory. Lecture 2-2 discusses a method to compute the mixed Nash equilibrium and an illustration of it in the context of law enforcement, and introduces evolutionary game theory.

Readings 2-1	Math Lab 2-1	Readings 2-2	Math Lab 2-2
Osborne 4.12, 4.1–4.5	Exercises	Osborne 4.6–4.10 Osborne 13.1	Part 1: Test 2 (Material: 1-2, 2-1) Part 2: Exercises

Week 3: Order

Week 3 is devoted to extensive games with perfect information. In Lecture 3-1, an extensive game is a game in which the order in which agents move is modelled explicitly. We discuss the relation between games in extensive form and normal form, the subgame perfect Nash equilibrium concept, as well as backward induction. In Lecture 3-2, we discuss several applications of extensive games.

Readings 3-1	Math Lab 3-1	Readings 3-2	Math Lab 3-2
Osborne 5	Exercises	Osborne 6.1, 6.3, 7.1, 7.3, 7.4, 7.7	Part 1: Test 3 (Material: 2-2, 3-1) Part 2: Exercises

Week 4: Information

Week 4 discusses game models where agents do not have full information about the game. In Lecture 4-1, we examine normal form games in which the agents lack information about one or more features of the game, such as the payoffs of the other players. In Lecture 4-2, we continue the discussion of the implications of imperfect information, but now for extensive games. The importance of signaling is described and we introduce the notion of a weak sequential equilibrium.

Readings 4-1	Math Lab 4-1	Readings 4-2	Math Lab 4-2
Osborne 9.1–9.3, 9.5, 9.7	Exercises	Osborne 10	Part 1: Test 4 (Material: 3-2, 4-1) Part 2: Exercises

Week 5: Cooperative Games

In this week we start with the part on cooperative game theory. In Lecture 5-1, we discuss the cooperative game model. A cooperative game describes a situation where agents (called players) can earn certain payoffs by cooperation. It assumes that players can make binding agreements. (In contrast with non-cooperative games where binding agreements are not possible.) In this lecture, we focus on the main solutions for cooperative games: the Core and the Shapley value. In Lecture 5-2, we discuss several applications of cooperative games, the Core and the Shapley value. Specifically, we discuss voting games and assignment/matching games (that are used in real life applications such as school choice, organ donation, housing and partner choice).

Readings 5-1	Math Lab 5-1	Readings 5-2	Math Lab 5-2
Osborne 8.1–8.3	Exercises	Osborne 8.5–8.8	Part 1: Test 5 (Material: 4-2, 5-1) Part 2: Exercises

Week 6: Bargaining and Externalities

In Lecture 6-1, we discuss bargaining models. Many agreements, trades etc. are realized through bargaining and negotiations. Examples are bargaining between: employees and employers, unions and employer organisations, governments, parents and children, etc. We discuss cooperative as well as non-cooperative approaches to bargaining. We also apply bargaining to find a strategic foundation of the Shapley value for cooperative games. In Lecture 6-2, the final lecture of this course, we analyze externalities in the context of so-called river sharing problems. In these problems we apply methods of cooperative as well as non-cooperative game theory to determine optimal allocation rules of scarce water resources, possibly by letting certain countries who consume water transfer a monetary compensation to other countries who abstain from water consumption.

Readings 6-1	Math Lab 6-1	Readings 6-2	Math Lab 6-2
Osborne 16	Exercises	Béal <i>et al.</i>	Part 1: Test 6 (Material: 5-2, 6-1) Part 2: Exercises

Week 8: Exam, date March 22

3. Readings

a. Textbook

Osborne, M. J. (2009), *An Introduction to Game Theory*, Oxford: Oxford University Press. (selected sections are mandatory)

b. Recommended reading week 6

Béal S., Ghintran A., Solal, P. and Rémila E. (2013), The River Sharing Problem : A Survey. *International Game Theory Review*, 15 , 1–19.

4. Assessment

Final Grade Calculation

Every week, there will be a test about the material covered in that week's first lecture/seminar and—except for week 1—the second lecture/seminar of the previous week. In Weeks 1 and 2 these tests will be done in the second seminar of the week. In Weeks 3, 4, 5 and 6, the tests will be given as take-home tests to be solved at home. The tests will be published on Tuesday after the seminar (around 12.45), and the answers should be submitted before the start of the Thursday seminar (before 11.00) of the same week through Canvas.

There are thus six weekly tests that you could take. Your total test score will be the average of your five best results. If you miss a test, then you score 0 for that test and your weekly test score will be the average of the remaining five tests. This total test score determines 50% of your grade. In Week 8, there will be a final exam covering all material and determining the other 50% of your grade.

John Stuart Mill College Marking and Resit Policy

The general guidelines on assessment and resits can be found in the John Stuart Mill College regulatory guidelines. If you have failed the course, then there will be one integrated resit covering all weekly tests and the final exam.

Plagiarism

Plagiarism of any kind will not be tolerated. Plagiarism not only involves the direct copying of existing texts without attribution, but also occur through sloppy citation and referencing and via self-plagiarism. Self-plagiarism occurs when students submit work twice in fulfilment of multiple degree requirements. Electronic detection software will be used to detect plagiarism. In submitting a text, the student implicitly consents to the text being entered into the database of the detection programme concerned. More information about plagiarism and its consequences can be found in the student handbook.